Investing in Commodities: from Roll Returns to Statistical Arbitrage

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Overview

Why Invest in Commodities?

- Traditional reasons: diversification of portfolio.

Why Invest in Commodities?
The Gurus

Why Invest in Commodities?
Inflation?

Types of Commodities Investments

- Broad indices: GSCI, DJ AIG, CRB, RICI. Very different return/volatility characteristics.
- Individual futures contracts.
- Spot investment. Small investor mostly restricted to ETFs with metal or rare earth holdings.

Equities Dream
Berkshire Hathaway

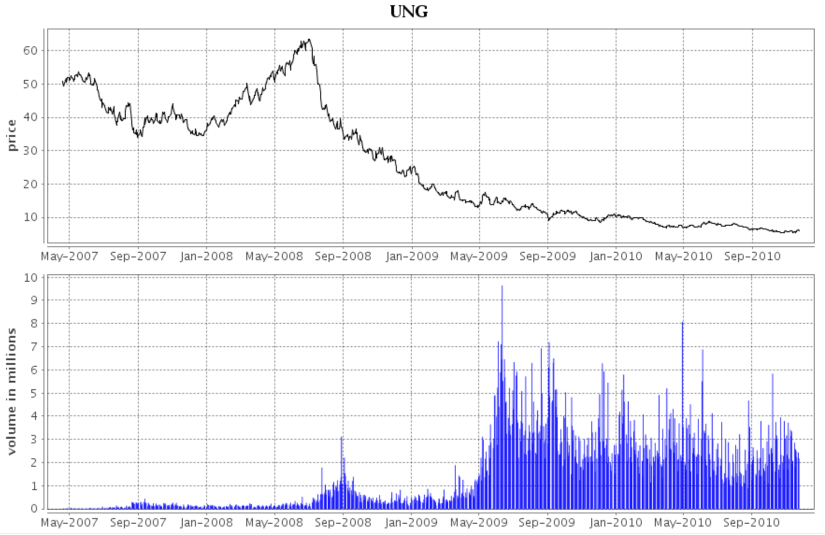
- 1965: share price around $18 per share.
- 2010: share price around $120,000 per share.
- $10,000 invested in 1965 is worth $66,666,667 today.
- Why didn't my grand-daddy invest $10,000 in Berkshire back in 1965?
- $1,000 invested in 1965 is worth $6,666,667 today.
- Why didn't my daddy invest $1,000 in Berkshire back in 1965?

Commodities: No Such Thing as Buy and Hold

- The spot commodity is generally perishable. We shall not consider here commodities such as gold or other easily storable metals.
- There is no direct analogy to market capitalization for commodities.
- Thus there is no agreed upon way to define the composition of the aggregate commodity futures market.
- *"Lacking a market capitalization based portfolio weighting scheme, commodity indices can best be thought of as commodity portfolio strategies." [2]*
- Every futures contract is born with an expiration date.
- Thus, investing in futures contracts implies considering roll-over strategies. Thus, every futures investment really depends upon the futures term structure.
- Mean reversion is an important feature of commodity prices.

- There is much to investigate with respect to indices, but we shall not pursue this here.
- Rather, we shall focus on trading in individual commodities futures.
- The most accessible way for a small investor to participate is through ETFs. Let's take a look at some examples.

Pitfalls of Naive Futures Investing



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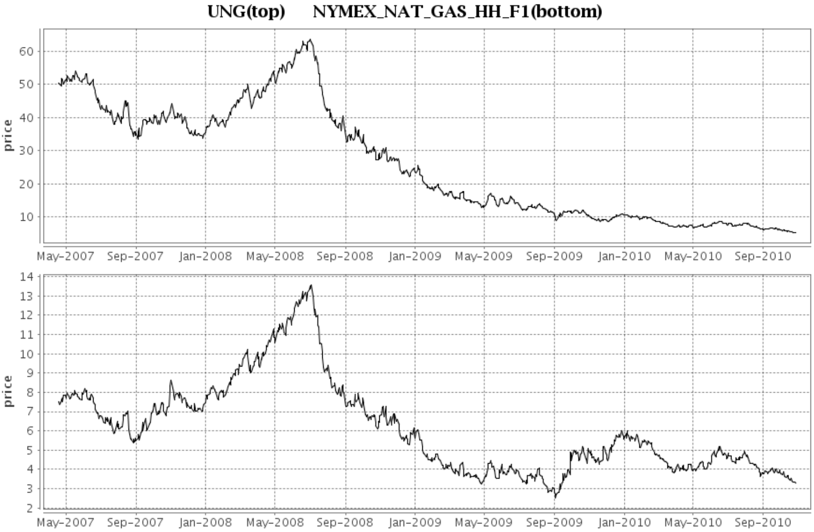

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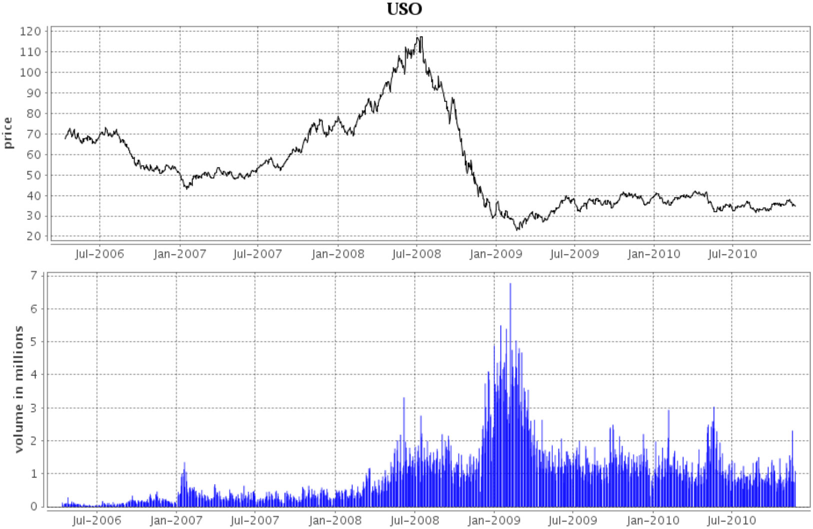

Pitfalls of Naive Futures Investing

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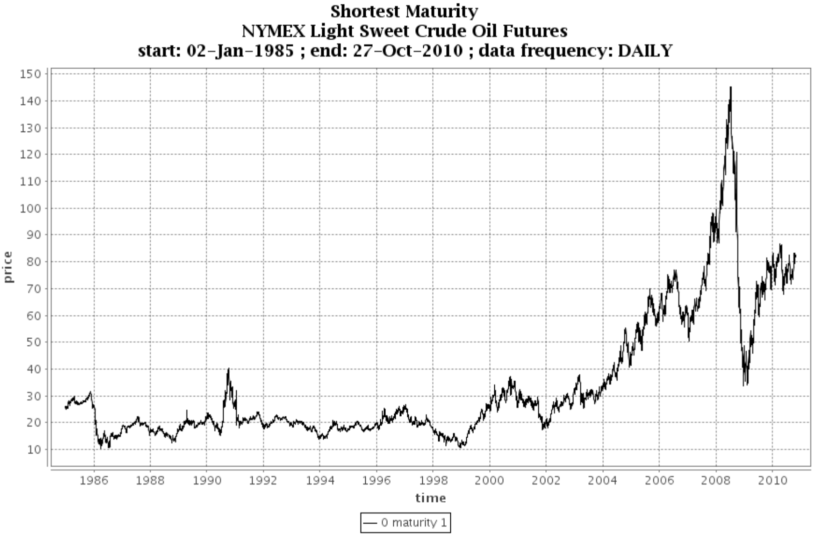


Roll Returns

The F1 series is deceptive because while it is a decent proxy for the spot price, it does NOT represent returns to an investor. This is because at each roll-over date, you leave one contract and enter another. So the PL has breaks.

Imagine rolling over the nearby futures contract. Say the prices at beginning of period i are PB_i and at the end of the period PE_i. Let us consider 3 periods.

- $PL = (PE_1 - PB_1) + (PE_2 - PB_2) + (PE_3 - PB_3)$
- This is not the same as the naive "spot PL": $PE_3 - PB_1$.
- The difference is in the so-called "roll PL": $PL = (PE_3 - PB_1) + (PE1 - PB_2) + (PE2 - PB3)$
- In this way, we write the total PL as a sum of the "spot PL" + "roll PL".

Roll Returns
Unlevered Returns

- Let us be more precise about the roll returns. First imagine a non-levered investment in, say, a stock. Let us suppose an investor has A dollars to invest, and the beginning stock price is p_1. Later, when the stock price is p_2, the stock is sold. We have that the investor buys $\frac{A}{p_1}$ shares, and the PL is

$$\frac{A}{p_1}(p_2 - p_1) = A\frac{p_2 - p_1}{p_1} = Ar_1 \tag{1}$$

where we define the return $r_1 \stackrel{\text{def}}{=} \frac{p_2 - p_1}{p_1}$. At the end of the transaction, the investor has an amount

$$A + \ PL = A + Ar_1 = A(1 + r_1) \tag{2}$$

Roll Returns
Unlevered Returns

- If the investor now takes all that money and invests again in a stock, with starting price p_3 and sells later at price p_4, the investor shall have an amount $A(1+r_1)(1+r_2)$, where $r_2 \stackrel{\text{def}}{=} \frac{p_4 - p_3}{p_3}$. Note that we can write
$$A(1+r_1)(1+r_2) = A\frac{p_2}{p_1}\frac{p_4}{p_3} \quad (3)$$
- It now clear how to extend the formula to further investments.

Roll Returns

Levered Returns

- Let us suppose now that an investor invests in futures. The investor starts with an amount of wealth A, as before. Let us suppose that the investor will roll over nearby futures, that the initial price of the nearby contract is p_1, the price of the same contract at the rolling date is p_2, the price of the next nearby futures contract at the rolling date is p_3, and the price of that contract at the following rolling date is p_4. In order to match the PL profile of a non-levered investment, the investor should commit $\frac{A}{p_1}$ to the position in the first contract, and keep the money A available for losses. In this way, the investor can sustain the maximum loss($p_2 = 0$) without incurring any debt. (In practice, the investor would be forced to submit a part of the money to a margin account held with a broker, and would invest the rest at the risk free rate). We shall ignore the effect of interest earned on this money while the futures contract is held.

Roll Returns

Levered Returns

- Now, the PL on the first futures contract is

$$PL = \frac{A}{p_1}(p_2 - p_1) = A\frac{p_2 - p_1}{p_1} = Ar_1 \tag{4}$$

where we define the return $r_1 \stackrel{\text{def}}{=} \frac{p_2 - p_1}{p_1}$. (In actuality, the contract is marked to market everyday, but the net effect is the same as the above).

- Thus, after the first contract, the wealth of the investor is $A + PL = A(1 + r_1) = A\frac{p_2}{p_1}$. Again, to mimic the PL profile of an unlevered investment, the investor shall commit $\frac{A\frac{p_2}{p_1}}{p_3}$ to the next nearby futures contract. Then the PL on the second contract is

$$PL = A\frac{p_2}{p_1}\frac{(p_4 - p_3)}{p_3} \tag{5}$$

- The wealth of the investor after the second contract ends is thus $A\frac{p_2}{p_1}\frac{p_4}{p_3}$, exactly as if it were an unlevered investment, as above.

Roll Returns
Levered Returns

- In this setting, we can again rearrange terms, and think of the returns as a spot return plus roll returns. Namely

$$A \frac{p_2}{p_1} \frac{p_4}{p_3} = A \frac{p_4}{p_1} \frac{p_2}{p_3} \tag{6}$$

  We see that by rearranging terms, we have a portion due to the return from the change in price from p_1 to p_4, which we can call the "spot return", and a portion due to the change in price from p_2 to p_3, which we can call the "roll return". (Note that the terminology "spot return" is more appropriate for a strategy rolling over nearby futures contracts. Perhaps it would be better to call it a "series return".)
- We see how this extends to further rolling over more futures contracts. We have thus decomposed the returns as one "spot return", and many "roll returns".

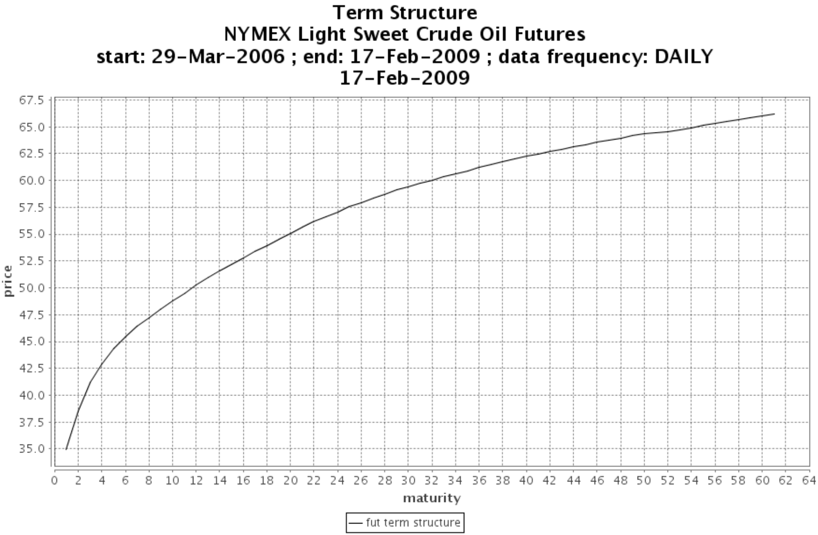
Roll Returns

- If the term structure is increasing with time to maturity(contango), then the roll returns are negative.
- If the term structure is decreasing with time to maturity(backwardation), then the roll returns are positive.
- Scenarios: imagine spot price constant, term-structure constant; then a rolled futures position consistently makes or loses money depending on whether there is backwardation or contango.
- At first sight it may seem counter-intuitive, but it is possible that the spot price of a commodity could go to infinity, while a rolled futures position always loses money! Conversely, spot prices can fall, but a rolled futures position can make money nonetheless.

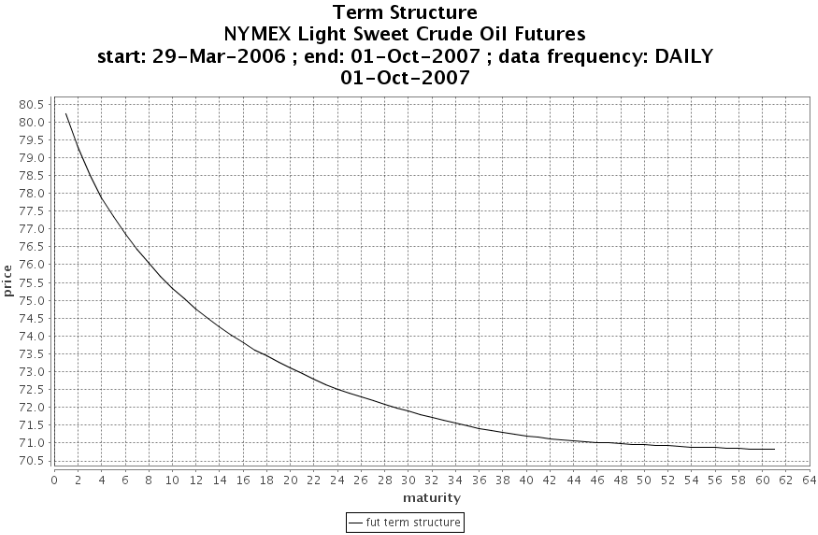
Roll Returns
Summary

- Returns on rolled futures positions depend on two main factors. One is the change of the spot price. The other is the shape of the term structure.
- For long rolled positions, backwardation helps, contango hurts.
- A rolled futures strategy which goes long in a contango market will only be profitable if the spot price rises *even more than* the effect of the contango.
- On the other hand, such a strategy in a backwardated market will always be profitable if the spot price rises at all, and can even be profitable if the spot price falls, depending on how much it falls.

Term Structure Contango



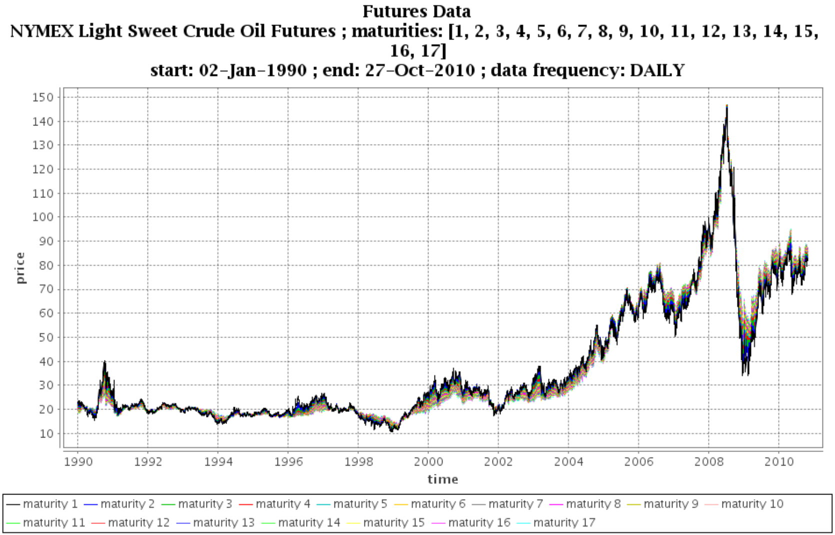
Term Structure Backwardation



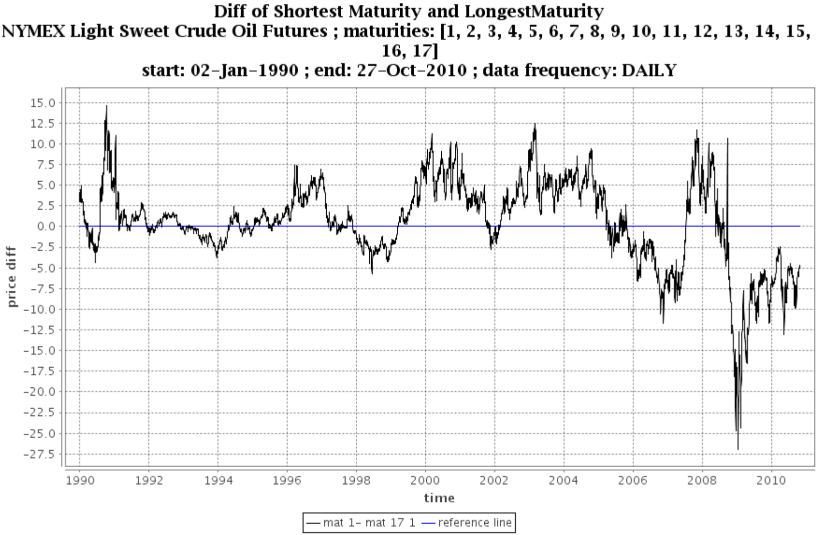
REMINDER TO THE SPEAKER

Play the term structure movie.

Periods of Backwardation



Periods of Backwardation



Some Strategies

- Oil: go long and wait for wars.
- Gas: go long and wait for hurricanes.
- Agriculture: go long and wait for droughts.
- Momentum strategy: go long backwardated contracts and short contangoed contracts with high volatility, [3].
- I have not personally checked these strategies .

Convenience Yield

- Benefits obtained from owning a commodity which are NOT obtained by holding a futures contract.
- Examples include the ability to keep a production process running, or to profit from supply disruptions.
- The theory of storage: dates to the 1930s and predicts that the return from purchasing a commodity at time t and selling it for delivery at time T through a futures contract should equal the interest foregone on the principal plus the storage costs and less the return given by the convenience yield.
- Low inventories and expectations of shortages tend to be associated with high convenience yields.

Modelling the Convenience Yield

- let $U(t)$ denote the instantaneous cost of storage (as a proportion of the spot price)
- similarly let $C(t)$ be the convenience yield
- denote by $\delta(t)$ the net convenience yield: $\delta(t) \stackrel{\text{def}}{=} C(t) - U(t)$
- we can then model the spot price of the commodity as:

$$\begin{aligned} dS =& \mu(S_t, t)dt + \sigma(S_t, t)dW_t + (U_t - C_t)S_t dt \\ dS =& (\mu(S_t, t) - \delta_t S_t)dt + \sigma(S_t, t)dW_t \end{aligned}$$

- No arbitrage or risk-neutral arguments imply:

$$F(t, T) = \widetilde{E}[e^{(\int_t^T (R(s)-\delta(s))ds)}]S_t \stackrel{simple\ case}{=} e^{(r-\delta)(T-t)}S_t \quad (7)$$

Simplistic Stories to go with the Math

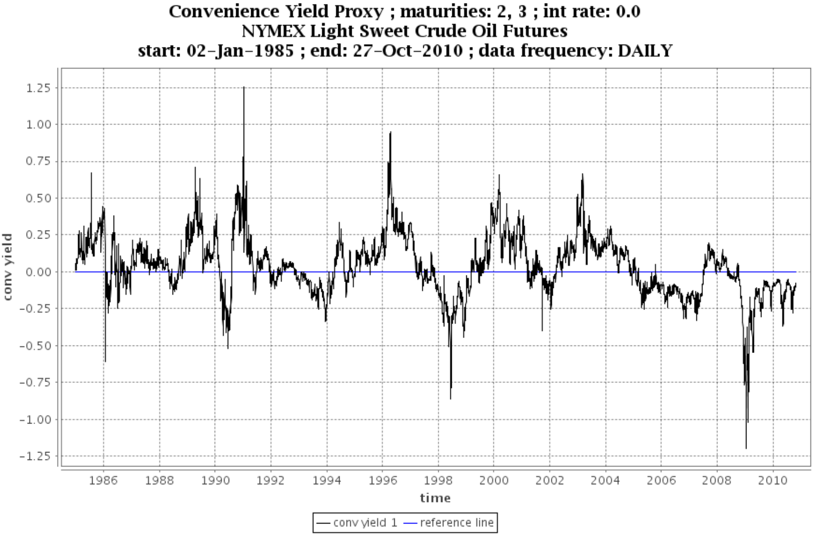
- Low supplies $\Longrightarrow$ cheap storage, high convenience yield
- The math says: U small, C big $\Longrightarrow \delta$ big $\Longrightarrow$ backwardation.
- The story says: those holding the commodity benefit from holding it, so they will only sell in the near term for higher prices.
- The converse “explains” contango.

Convenience Yield Proxy

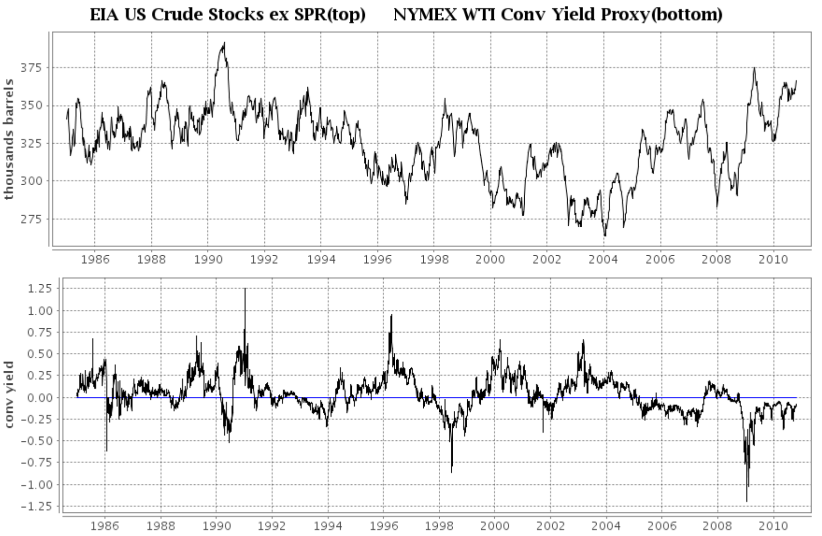
- Solving the equation above for δ yields a proxy for δ.
- For example, one may take T_1 and T_2, the two nearest expirations, and form the series

$$\delta = r - 12 \log \left(\frac{F(t, T_2)}{F(t, T_1)} \right) \tag{8}$$

Convenience Yield



Convenience Yield



Gibson Schwartz Model

- One factor models of commodities don't capture the term structure well because they imply that all maturities are perfectly correlated, which is contradicted by the data.
- For this reason, plus the time series properties of the convenience yield proxy, plus the economic reasons for a mean-reverting convenience yield, the following two-factor model was proposed by Gibson and Schwartz.

$$dS = (\mu - \delta)S_t dt + \sigma_1 S_t dW_1 \tag{9}$$

$$d\delta = \kappa(\theta - \delta)dt + \sigma_2 dW_2 \tag{10}$$

$$dW_1 dW_2 = \rho dt \tag{11}$$

Schwartz Smith Model

- The Gibson Schwartz model is pretty cool, but a (possible) disadvantage is that the spot price and the convenience yield are generally very highly correlated. Also, the convenience yield tends to get a lot of flack and is considered to be a rather elusive concept.
- The Schwartz Smith model may be obtained from the Gibson Schwartz model via an affine transformation. The two models are equivalent. In the Schwartz Smith model, the two factors tend to be more orthogonal, and are intuitively interpreted as a long-run evolution together with a short-term process which mean-reverts to the long-run process. It looks like this:

$$\begin{aligned}
log(S_t) &= x_1 + x_2 \\
dx_1 &= \mu dt + \sigma_1 dW_1 \\
dx_2 &= -\kappa x_2 dt + \sigma_2 dW_2 \\
dW_1 dW_2 &= \rho dt
\end{aligned}$$

Beyond Schwartz Smith

- There are models incorporating stochastic volatility and/or jumps. We shall not discuss them here.
- There are also models that take the term structure as the primary object, a la HJM.
- But to continue on the path outlined above, the next thing to do is to add more factors to our Gaussian model.
- The Schwartz Smith model may be extended by adding more Ornstein-Uhlenbecks. This was studied by Schwartz and Cortazar, and Cortazar and Naranjo. In general, we can look at affine term structure models (a la Dai and Singleton, Vasicek, Langetieg, et al in the fixed-income literature). In particular, we shall consider the following types of models.

Affine Term Structure Models

Let S_t denote the spot price. Let X_t be a vector of state variables, and let w denote a constant vector of weights. We write

$$log(S_t) \stackrel{\text{def}}{=} w^T X_t = w_1 X_1(t) + w_2 X_2(t) + \cdots + w_n X_n(t)$$

We specify the dynamics of X as

$$dX_t = (AX_t + b)dt + RdW_t$$

Here A is a matrix, b is a vector, W is a multi-dimensional brownian motion, and we specify its covariance as $\Sigma \stackrel{\text{def}}{=} RR^T$. We shall assume that A, b, and R are constant matrices for now. The solution to this SDE is

$$X_t = e^{At}[X_0 + \int_0^t e^{-As} bds + \int_0^t e^{-As} RdW_s]$$

Affine Term Structure Models

This is multi-variate normal with mean

$$e^{At}X_0 + \int_0^t e^{A(t-s)} b\,ds$$

and covariance matrix

$$\int_0^t e^{A(t-s)} \Sigma (e^{A(t-s)})^T ds$$

Affine Term Structure Models

We assume a constant market price of risk vector λ, and under change of measure, the risk-neutral dynamics become

$$dX_t = (AX_t + b - \lambda)dt + Rd\widetilde{W}_t$$

This has the same form as above, so futures prices become

$$F(t,T) = \widetilde{E}[S_T|\mathcal{F}_t] = e^{[\widetilde{E}[log(S_T)|\mathcal{F}_t] + \frac{1}{2}\widetilde{V}[log(S_T)|\mathcal{F}_t]]}$$

This is trivially computed using the above formulas, for we have

$$\begin{aligned}\widetilde{E}[log(S_T)|\mathcal{F}_t] =& w^T\widetilde{E}[X_T|\mathcal{F}_t]\\ \widetilde{V}[log(S_T)|\mathcal{F}_t]] =& wCov(X_T)w^T\end{aligned}$$

Affine Term Structure Models
Calibration

- It follows that the log of futures prices are linear functions of the state variables for such a model. Since the state variables are generally unobservable, a useful method for calibration of the model is the Kalman filter.
- The Kalman filter estimates the model parameters, and also the unobserved values of the state variables.
- The Kalman filter easily accomodates uneven time series data panels, missing observations, and provides standard errors for the estimates.
- For fixed model parameters, the Kalman filter can update estimates of state variables in real time(online), since the algorithm is recursive.
- A problem with the Kalman filter is that it is mostly useful for Gaussian models.

Schwartz Smith 3 and 4

We now specialize to the models studied by Schwartz alone or in collaboration with Gibson, Smith, Cortazar. Here we take w to be a vector of ones, and

$$A = \begin{bmatrix} -\kappa_1 & 0 & \dots & 0 \\ 0 & -\kappa_2 & \dots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & \dots & -\kappa_n \end{bmatrix} \qquad b = \begin{bmatrix} \mu \\ 0 \\ \vdots \\ 0 \end{bmatrix} \tag{12}$$

and $\Sigma = DCD^T$, where C is a correlation matrix $C_{i,j} = \rho_{ij}$ and

$$D = \begin{bmatrix} \sigma_1 & 0 & \dots & 0 \\ 0 & \sigma_2 & \dots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & \dots & \sigma_n \end{bmatrix} \tag{13}$$

Schwartz Smith 3 and 4

- The Schwartz Smith model is obtained by exogenously imposing $\kappa_1 = 0$ in the model above.
- I shall call Schwartz Smith 3(4) the model above with 3(4) factors and $\kappa_1 = 0$. In these models, the log of the spot is like a BM factor, with an OU mean-reverting to it, with an OU mean-reverting to it, with an OU mean-reverting to it, ...
- Fundamental difference with interest rate models is that the spot is non-stationary, due to the BM factor.
- Variations are to change the first BM factor to: i) OU(pure mean-reverting model) ii) OU with deterministic drift
- Calibrating the pure mean-reversion model for oil or gas after 2000, the mean-reversion rate in the first OU generally goes to zero, so there is little difference with the Schwartz Smith model.

Schwartz Smith 3 and 4
Motivation

Why extend the Schwartz Smith model to more factors?

- Get better fits to the futures term structure, in particular much better fit the to term structure of volatilities. (However, this is not to say that Schwartz Smith 2 does not already do a pretty good job).
- PCA. Reminder to speaker: run PCA

Statistical Arbitrage

- Since our mantra is that every futures investment is a strategy, we might consider it worth our time and efforts to investigate along the lines of statistical arbitrage.
- We have indeed explored several such strategies. Our results look rather promising.

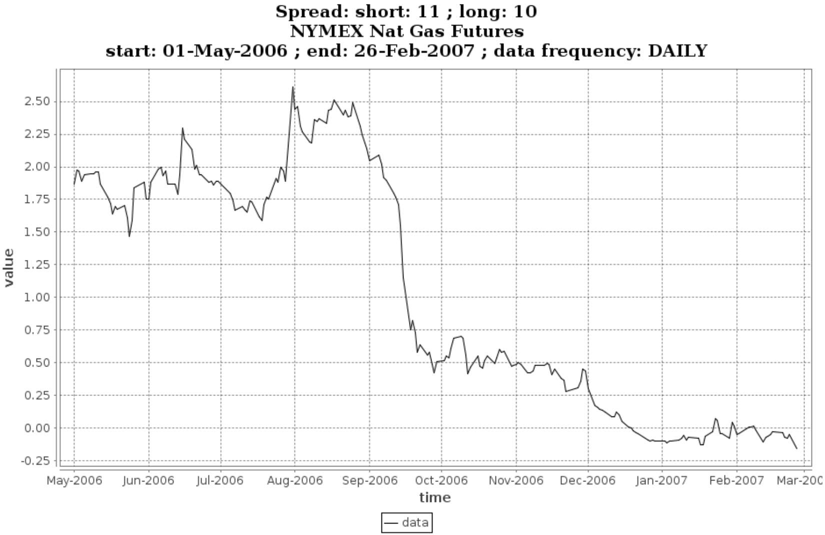
Natural Gas Trading

- Amaranth. This was a very large and successful hedge fund based in Greenwich, CT. On September 18th, 2006, the founder of Amaranth informed investors that the fund had lost an estimated 50% of their assets month-to-date. By the end of September 2006, the losses amounted to $6.6 billion, making it the largest hedge fund collapse in dollar terms. It was all because of natural gas futures.
- In June 2007, the U.S. Senate Permanent Subcommittee on Investigations(PSI) released a report on "Excessive Speculation in the Natural Gas Market", [1]. The PSI subpoened trading records from NYMEX and other sources. As a result, there exists a public record of Amaranth's trades. During 2006, Amaranth had large positions in spreads that were of the form: long winter, short summer. Two of the largest spreads were: long JAN 2007, short NOV 2006, and long MAR 2007, short APR 2007.

Natural Gas Trading

- MotherRock. According to the PSI report, the hedge fund MotherRock believed the MAR/APR 2006 spread to be overpriced and shorted it(short MAR, long APR). In the summer of 2006, it is alleged that Amaranth made such large purchases of the spread that they artificially pushed up its price; so much so that MotherRock could not maintain its margin requirements. The hedge fund folded.
- Saracen. In February of 2008, the hedge fund Saracen lost somewhere between $400-800 million on the MAR/APR natural gas spread. They were betting the spread would fall, but instead it increased significantly.

The Spread that Sank Amaranth



REMINDER TO THE SPEAKER

Thank the audience.

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